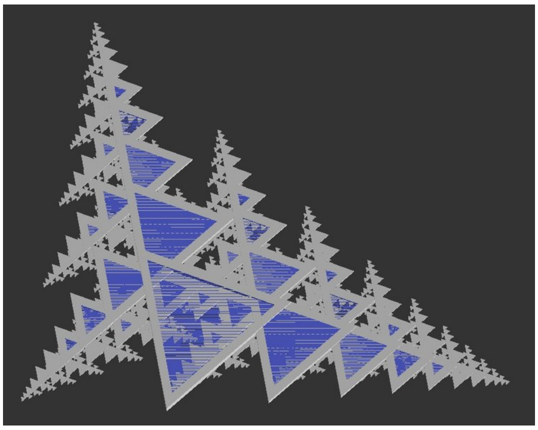

FIGURE 1. An example of an iterated function system (IFS) fractal

FRACTALS, 7.5 HP

JULIE ROWLETT

1. Course logistics

- **Dates:** November 5, 2018 – December 28, 2018
- **Last day for application:**
- **Examiner and email address for applications:** Julie Rowlett, julie.rowlett@chalmers.se
- **Responsible department:** Mathematics Department

2. Course description

The term fractal was first used by Benoit Mandelbrot in 1975. Since that time fractals have fascinated both researchers and the general public. The general notion of fractals can be traced back to the 17th century, and many important examples of fractals arised through the study of complex dynamics at the turn of the 20th century. Our study of fractals begins with an introduction to measure theory focusing on the construction of the Hausdorff measure. We will continue to use Hausdorff measure to define the notion of Hausdorff dimension; one way to define a fractal is via Hausdorff dimension. Next, we will specialize to a specific type of fractals known as IFS (iterated function system) fractals. Examples include Cantor sets and the Sierpinski gasket; these are only two of presumably at least countably infinitely many more such examples. We will see how to compute the Hausdorff dimension of IFS fractals. In the second half of the course we will focus on complex dynamics. Indeed, the famous Mandelbrot set arises from the study of complex dynamics. We will see how fractals materialize out of the study of holomorphic dynamics and the iteration of rational functions. At the end of the course we will discuss the current state-of-the-art and open research problems.

The prerequisites for this course are: analysis in one and several variables together with complex analysis. It may therefore be appropriate for suitably prepared and motivated master's students. In case of questions, please contact the examiner.

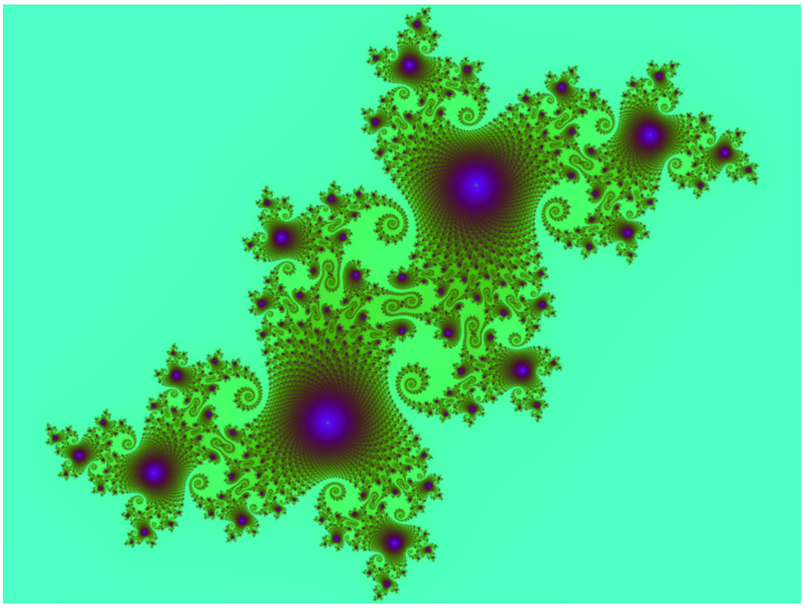

FIGURE 2. An example of a fractal arising from complex dynamics as the Julia set of a rational function

3. Course plan

We shall have eight weeks structured as follows.

3.1. Measure theory.

3.2. Hausdorff measure and dimension.

3.3. IFS fractals.

3.4. Holomorphic dynamics: more fractals.

3.5. Fixed points.

3.6. Complex dynamics of rational functions.

3.7. Fatou and Julia sets of rational functions and fractals.

3.8. Further directions.

4. Literature

- *Real analysis,* by Gerald Folland.
- *Complex Dynamics* by Carleson & Gamelin.
- Typed lecture notes will also be available.

5. Fractals, 7.5 hp, Official Information

(1) **Confirmation:** The syllabus was confirmed by the Head of the Department of XXX 200X-XX-XX, 200X-XX- XX.
Disciplinary domain: Science
Department in charge: Department of Mathematical Sciences.
Main field of study: Mathematics

(2) **Position in the educational system:** Elective course; third-cycle education.

(3) **Entry requirements:** Analysis in one and several variables, complex analysis.

(4) **Course content:** In this course we will: (1) introduce the notions of sigma algebras and fundamental concepts from measure theory; (2) construct the Hausdorff measure, relate it to the Lebesgue measure, and define Hausdorff dimension; (3) introduce iterated function system fractals and demonstrate fundamental properties about them including the calculation of their Hausdorff dimension; (4) introduce the notions of complex dynamics and demonstrate fundamental results in complex dynamics including how fractal sets arise in complex dynamics; (5) connect these topics to current research areas and physics to as great of an extent as is possible.

(5) **Outcomes:** After the completion of this course the Ph.D. student is expected to be able to:
- Understand Hausdorff measure and dimension;
- Define an iterated function system fractal and compute its Hausdorff dimension;
- Understand Fatou and Julia sets of holomorphic functions and complex dynamics.

(6) **Required reading:** During the course lecture notes will be compiled in a collective process involving both the students and the examiner. Supplementary reading material is listed in the bibliography below.

(7) **Assessment:** At the end of the course, students will be free to choose between two options:
(a) an oral exam, or
(b) a written take-home exam.

A Ph.D. student who has failed a test twice has the right to change examiners, if it is possible. A written application should be sent to the Department.

In cases where a course has been discontinued or major changes have been made a Ph.D. should be guaranteed at least three examination occasions (including the ordinary examination occasion) during a time of at least one year from the last time the course was given.

(8) **Grading scale:** The grading scale consists of VG or U.

(9) **Course evaluation:** The course evaluation is carried out together with the Ph.D. students at the end of the course, and is followed by an individual, anonymous survey. The results and possible changes in the course will be shared with the students who participated in the evaluation and to those who are beginning the course.

(10) **Language of instruction:** The language of instruction is English.

(11) **Course Schedule:** TBD.

(12) **Literature:** *Real analysis,* by Gerald Folland. *Complex Dynamics* by Carleson & Gamelin. Typed lecture notes will also be available.